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Systematic Modeling of Sludge Filtration Process Using Dimensional Analysis Technique

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ABSTRACT
The handling and disposal of sludge; the largest constituent removed in the process of treating wastewater is one of the greatest challenges facing the environmental engineer. The sludge has high water content and compressibility attribute and as such it is expedient to dewater it to reduce its volume and prevent environmental health hazard. This study presents a sludge filtration equation which incorporates the compressibility coefficient using a modified dimensional analysis technique. The equation was validated using experimental data from a pilot scale sand drying bed equipment yielding a close agreement between the theoretical and experimental values of the slope and intercept with a correlation coefficient ranging from 0.94- 0.98. The experimental slope and intercept was found to be (1260913.48 s/m^6 , 4872.53 s/m^3) (5359604.57 s/m^6, 844882.56 s/m^3), (112117050.4 s/m^6, -2135816.16 s/m^3), (145562880 s/m^6, -30497917.03 s/m^3) while the theoretical slopes and intercepts are (1257426.75 s/m^6, 5270.26 s/m^3),(4579418.42 s/m^6, 905658.24 s/m^3), (112117075 s/m^6, -21358166.74 s/m^3), (206699290.5 s/m^6, -4589555.58 s/m^3) respectively. The equation also accounts for the compressibility attribute believed to affect sludge filtration process.
KEYWORDS: Sludge, Drying bed, Filtration, Compressibility, Dimensional analysis.

I. INTRODUCTION

The treatment of wastewater before disposal is an important step in the minimization of environmental pollution. In the process of treating wastewater, sludge, which is amongst the constituents removed, is by far the highest in volume. The handling and disposal of this sludge is one of the greatest challenges facing the environmental engineer. The sludge has high water content and compressibility attribute and as such it is expedient to dewater it to reduce its volume and prevent environmental health hazard.

The dewatering of sludge using the constant vacuum filtration method has been adopted in full scale in the 1920s. Since then, a number of equations have been presented by various contributors aimed at improving the performance of the sludge filtration process, Carman, (1934), (1938); Race, (1935); Coackley, (1956); Ademiluyi and others, (1987), (1984), (1990); Gale, (1975); Anazodo, (1974); Ademiluyi, (1986). However, their research was limited to experimental work which could not provide an insight into the interactive nature of sludge filterability.

The application of $FMTL_XL_YL_Z$ dimensional technique to sludge filtration using sand dying bed is an improvement to sludge filtration theory which became necessary because previous equations describing filtration process using constant vacuum filtration has been criticized for lack of agreement between theory and experiment, Ademiluyi, (1983). Also, the dimensional analysis equation presented by Anazodo (1974) which was based on the constant vacuum approach has also been criticized partly because the equation did not account for the compressibility attribute of sludge believed to affect sludge filtration process. Sludge drying bed filtration is one of the earliest processes used in the dewatering of sludge before the introduction of mechanical processes. The waste can be dewatered naturally in an open or covered sand bed under variable pressure and also requires a large amount of land for its operation. Sand drying bed is affected by such uncontrollable factors as Rainfall, Humidity and Temperature. The process is cost effective and easy to operate than the mechanical system and usually produces sludge cake of about 25-40% solid.

Hence, the objective of this research is to present a sludge drying bed filtration equation which incorporates the compressibility coefficient believed to affect sludge filtration process using a modified dimensional analysis technique.

II. MATERIAL AND METHODS

2.1 FILTRATION APPARATUS

The pilot scale drying bed apparatus is a rectangular structure of dimension 120cm x 75cm x 80cm which requires 0.27cm^3 of the sludge sample for filtration experiment. The filter unit consists of gravel and sand with a drying area of 120cm long by 75cm wide. The empty apparatus is initially filled with gravel to a height of 20cm and then this column of gravel provides support for a column of sand of depth 20cm. The sludge is introduced to the dry bed by a bucket into the sand column and flows through the sand and gravel to drain out from a drain pipe at the base of the unit. Measuring tape is attached to the side of the apparatus to read directly the fall in height of the sludge at 2hour intervals. A 1000cm^3 measuring cylinder placed at the base of the unit collects the filtrate, were temperature of the liquid can be measured.

2.2 EXPERIMENTAL PROCEDURE

Digested sludge of 0.27m^3 volume collected from the imhoff tank of the University of Nigeria Nsukka was poured into the drying bed to a height of 30cm from the sand column. Before the filtration started, a known volume of the well stirred filtering sludge was collected and taken to the laboratory to enable the determination of the initial solid content of the sludge. The filtration started after 2minutes was allowed for sludge to settle at the top of the sand column so that the sludge provides the resistance (specific resistance) to flow of liquid after which the fall in height of the sludge with time and the corresponding change in pressures were recorded. Also the temperature of the sludge and the filtrate before and after filtration were recorded and was used to compute the density and the dynamic viscosity of the sludge.

The volume of filtrate collected into the cylinder was recorded for every 2hours interval for the first day of filtration and 24hours for subsequent days until the filtration came to an end. At the end of the filtration period, the specific resistance was determined using the modified equation.

The effect of different weight of ferric chloride dosage on the specific resistance based on the equation was investigated by taken five portions of 0.063m^3 of sludge and mixing it with various amount of ferric chloride dosage (10g,20g,30g,40g,50g) with distilled water(1liters,2liters,3liters,4liters,5liters). The mixture was stirred vigorously and poured into the drying bed and another filtration started. After the end of each of the separate five filtration experiments the specific resistance was found using the data collected. Also, at this point, a known volume of the wet sludge was taken to the laboratory to measure the compressibility coefficient of the sludge using the odometer test.

2.3.1 DIMENSIONAL ANALYSIS AND THE FMTL$_X$L$_Y$L$_Z$ TECHNIQUE

Dimensional analysis is a technique that involves the study of dimensions of physical quantities. It is used primarily as a tool for obtaining information about physical systems too complicated for full mathematical solution to be feasible. It enables one to predict the behavior of large systems from a body of small scale models. It affords a convenient means of checking mathematical equation. The technique of dimensional analysis has several important applications. It can be applied to the derivation of unknown parameters in the relationship between various variables and also in the analysis of model design. The FMTL$_X$L$_Y$L$_Z$ dimensional system is derived by the combination of MLT and FLT fundamental systems (that is, Mass, Length and Time/ Force, Length and Time). The length L in this system is differentiated into L$_X$L$_Y$L$_Z$ mutually perpendicular axes in space.

2.3.2 APPLICATION OF FMTL$_X$L$_Y$L$_Z$ DIMENSIONAL ANALYSIS TECHNIQUE

The application of FMTL$_X$L$_Y$L$_Z$ dimensional analysis technique to the derivation of unknown relations depends upon the concept of completeness of equation. A physical equation is the relationship between two or more physical quantities. Any correct equation expressing a physical relationship between quantities must be dimensionally homogeneous and numerically equivalent. Dimensional homogeneity states that every term in an equation when reduced to fundamental dimensions must contains identical power of each dimension. Applying the above principle to the derivation of filtration equation we can represent the relationship between volume of filtrate and the variables affecting sludge filtration process by:

$$V = A^a P^b \mu^c M^d t^e R^f \quad (1)$$

Were V is the volume of filtrate, A is the area of filtration, P is the variable pressure applied during filtration, μ is the dynamic viscosity of the fluid, M is the concentration of the sludge , t is the time of filtration and R is the specific resistance of the sludge. The superscripts a, b, c, d, e, f are unknown quantities to be determined.

For dimensional homogeneity we have that,

2.3 DERIVATION OF EQUATION

$$L_x L_y L_z = (L_x L_y)^a (F L_x^{-1} L_y^{-1})^b (F T L_z^{-2})^c (M L_x^{-1} L_y^{-1} L_z^{-1})^d (T^e)(M^{-1} L_z)^f \quad (2)$$

Were a, b, c, d, e, f are unknowns to be determined using dimensional homogeneity between variables.

For condition on:

$L_x L_y$: 1 = a- b - d (3)

L_z : 1 =-2c - d + f (4)

M : 0 = d - f (5)

F : 0 = b + c (6)

T : 0 = c + e (7)

From the above we obtained five simultaneous equations (equation 3- 7) in six unknown (a, b, c, d, e, f) and which was solved using normal equation to yield the following: a = d+3/2, b = ½, C = -1/2, d, e = ½ , f = d.
Substituting the evaluated values above,

$$V = A^{d+\frac{3}{2}} P^{\frac{1}{2}} \mu^{-\frac{1}{2}} M^{d} T^{\frac{1}{2}} R^{d} \quad (8)$$

$$V = (AMR)^{d} \left(\frac{A^{3}Pt}{\mu}\right)^{1/2} \quad (9)$$

$$V^{2} = (AMR)^{2d} \cdot \frac{PtA^{3}}{\mu} \quad (10)$$

$$\frac{V^{2}}{t} = \frac{A^{3}p}{\mu} \quad (AMR)^{2d} \quad (11)$$

Dividing through by V

$$\frac{V}{t} = \frac{A^{3}p}{V\mu} \quad (AMR)^{2d} \quad (12)$$

Inverting equation (12) we have

$$\frac{t}{v} = \frac{V\mu}{PA^{3}} \cdot (AMR)^{-2d} \quad (13)$$

A plot of $\frac{t}{v}$ versus M gave a straight line from which d = -1/2 and hence:

$$\frac{t}{V} = \frac{V\mu MR}{PA^{2}} \quad (14)$$

To accounting for compressibility coefficient in equation (14), we know that
Pressure (P) = ρgH were ρ is the density of water, g is the gravitational acceleration, and H is the height of sludge at a particular time interval.
Also $H = H_s + \Delta H$ were H_s is the initial heights of sludge and ΔH is the change in sludge height between two successive time periods. Hence pressure P = $\rho g(H_s + \Delta H)$
Substituting the above transformation into equation (14) we have:

$$\frac{t}{V} = \frac{\mu MRV}{A^{2}\rho g(H_s + \Delta H)} \quad (15)$$

Inverting equation (15) we have

$$\frac{V}{t} = \frac{A^{2}\rho gH_s + A^{2}\rho g\Delta H}{\mu MRV} \quad (16)$$

$$\frac{V}{t} = \frac{A^{2}\rho gH_s}{\mu MRV} + \frac{A^{2}\rho g\Delta H}{\mu MRV} \quad (17)$$

We can also express M, the initial solid content as $\frac{W_d}{V_{Sl}}$ were V_{Sl} is the volume of sludge and W_d = weight of dry sludge.
and ρgΔH = Δp ,
ρgHs = P_1 were P_1 is the pressure at the initial height of sludge.
Substituting the above transformation into equation (17) we have,

$$\frac{V}{t} = \frac{A^{2}P_1V_{Sl}}{V\mu W_d R} + \frac{A^{2}\Delta PV_{Sl}}{V\mu W_d R}$$

(18)

Dividing equation (18) through by A we have,

$$\frac{V}{At} = \frac{A^{2}P_1V_{Sl}}{AV\mu W_d R} + \frac{A^{2}\Delta PV_{Sl}}{AV\mu W_d R} \quad (19)$$

The volume of sludge V_{Sl} can be expressed also as $\frac{W_d}{P_S S_{Sl} \rho_w}$

Were P_S = percentage of solid content expressed in decimal, S_{Sl} = Specific gravity of sludge and ρ_w = density of filtrate.
Substituting the volume of sludge V_{Sl} with $\frac{W_d}{P_S S_{Sl} \rho_w}$ into equation (19) gives,

$$\frac{V}{At} = \frac{A^{2}P_1w_d}{AV\rho_w\mu S_{Sl} P_S W_d R} + \frac{A^{2}\Delta PW_d}{AV\rho_w\mu S_{Sl} W_d P_S R} \quad (20)$$

$$\frac{V}{At} = \frac{A^{2}P_1H_s}{V\mu P_S W_d R} + \frac{A^{2}\Delta PH_s}{V\mu W_d P_S R} \text{(Were } H_s = \frac{W_d}{A S_{Sl}\rho_w}\text{)} \quad (21)$$

$$\frac{V}{At} = \frac{A}{V}\frac{AP_1H_s}{\mu P_S W_d R} + \frac{A}{V}\frac{A\Delta PH_s}{\mu W_d P_S R} \quad (22)$$

$$\frac{V}{At} = \frac{A^{2}P_1H_s}{V\mu P_S W_d R} + \frac{A}{\mu W_d P_S R}\frac{\Delta P}{\Delta e} \quad \text{(Were } \frac{A}{V} X H_s = 1/\Delta e\text{)} \quad (23)$$

$$\frac{At}{V} = \frac{V\mu P_S W_d R}{A^{2}P_1H_s} + \frac{\mu W_d P_S RS}{A} \quad (24)$$

$$\frac{t}{V} = \frac{V\mu P_S W_d R}{A^{3}P_1H_s} + \frac{\mu W_d P_S RS}{A^{2}} \quad (25)$$

Equation (25) can be written in the form
t/v = bV + C (26)
Which represents an equation of a straight line with slope, b = $\frac{\mu W_d RP_s}{P_1H_sA^3}$ and intercept, C = $\frac{\mu W_d RP_sS}{A^2}$
and V is the volume of filtrate at a given time, t is the time of filtration.

III. RESULTS AND DISCUSSION

The results of experiments conducted during the sludge filtration experiments are all shown in tables 1 - 6. Table 1 shows the data for experimental slope and intercept used for verifying the modified equation. Table 2 shows the data used for the calculation of theoretical slope and intercept that was used to compare with the experimental value. Table 3 shows the result of the effect of using 10g of Ferric chloride dosage on specific resistance of the sludge and Table 4 shows the variation of specific resistance with the different weights of Ferric chloride dosage. Table 5 shows the laboratory result used in the determination of void ratio of the conditioned sludge with 10g weight of Ferric chloride dosage. Table 6 shows the data for computation of the sludge compressibility coefficient of the sludge.

Fig 1 shows the graph of the experimental and theoretical data used to validate the equation. From the graph, the slope and intercept for the experimental result is 1260913.48s/m^6 and 4872.52s/m^3 while that for the theoretical result is1257424.31s/m^6 and 5270.54 s/m^3 respectively and shows a close agreement between experimental and theoretical values for the periods of filtration tested. The modified equation predicts that the plot of t/v versus v is a straight line and from the graph the two plots shows a straight line with high coefficient of correlation of 0.98.

In figures 2-4 there was also a close agreement between theoretical slopes and intercept and the experimental slopes and intercept except in figure 2 were we had a small difference between experimental intercept and theoretical intercept. The theoretical intercept was higher than the experimental intercept by 7.2%. The difference may be attributed to more of the sludge particles been suspended and so the pressure of the fluid passing through the mass of settled sludge increased thereby allowing more filtrate to pass through and hence the ratio t/v reduced.

t(s)	V(m^3)	t/v (s/m^3)	V^2(m^6)	v.t/v(s)
0	0	0	0	0
7200	0.0717	100,488.49	0.0051	7200
14400	0.1034	139,264.99	0.0107	14400
21600	0.1283	168,421.05	0.0165	21600
28800	0.1505	191,362.12	0.0227	28800
36000	0.1709	210,649.50	0.0292	36000
43200	0.1805	239,335.18	0.0326	43200
∑	**0.8053**	**1,049,521.33**	**0.1168**	**151200**

Table 1: Data from filtration experiment used to calculate experimental slope and intercept

Volume of sludge (V) = 0.27 m^3, Initial Height of sludge (H_s) = 0.3m, Average Height of sludge (H) = 0.155m, Initial hydrostatic pressure (P_i) = 2931.9 N/m^2, Average Applied pressure (P_{av}) ρgh = 1514.82 N/m^2 , Average Temp. = 26^{o}c, Density of water = 996.23kg/m^3, Area (A) = 0.9 m^2, Dynamic Viscosity μ = 0.892N.s/m^2, Weight of dry solid W_d = 0.0157 kg, Initial Solid Content (M) = 0.058 kg/m^3, Percentage of solid expressed in decimal P_s = 0.05, Specific gravity of sludge S_{sl} = 1.05, Acceleration due to gravity (g) = 9.81 m/s^2,

From Regression Analysis, we know that the slope b is given by the formula below

$$b = \frac{n\sum v.\frac{t}{v} - \sum v \sum\frac{t}{v}}{n\sum v^2 - (\sum v)^2}$$

$$= \frac{7 X 151200 - 0.8053 X 1049521.33}{7X0.1168 - 0.6485}$$

$$= \frac{1058400 - 845179.53}{0.8176 - 0.6485}$$

$$= \frac{213220.47}{0.1691} = 1260913.48 \text{ s/m}^6$$

$$C = \frac{\frac{t}{v}}{n} - b\frac{v}{n}$$

= 149931.62 - 145059.09 = 4872.53 s/m^3

Solving for the specific resistance and the compressibility coefficient of the sludge we have from

$$\frac{t}{v} = \frac{V\mu W_d R P_s}{P_I H_s A^3} - \frac{\mu W_d R P_s S}{A^2}$$

$$R = \frac{A^3 P_I b H_S}{\mu W_d P_S} = \frac{0.9^3 X 2931.9 X 0.3 X 1260913.48}{0.892 X 0.0157 X 0.05 X} = 1.154645622 X 10^{12} \text{ m/Kg}$$

$$S = \frac{CA^2}{\mu W_d P_S R} = \frac{4872.53 X 0.9^2}{0.892 X 0.0157 X 0.05 X 1.154645622 X 10^{12}} = 0.000005 \text{ m}^2\text{/KN.}$$

V(m^3)	t/v(s/m^3)	V^2(m^6)	V.t/v(s)
0	0	0	0
0.0717	100780.26	0.0051	7225.94
0.1034	138372.02	0.0107	14307.67
0.1283	167899.93	0.0165	21541.56
0.1505	194226.02	0.0227	29231.02
0.1709	218417.56	0.0292	37327.56
0.1805	229801.81	0.0326	41479.23
0.8053	**1049497.6**	**0.1168**	**151112.98**

Table 2: Data used for calculation of theoretical slope and intercept

The dimensional equation is of the form **t/v** = b**v** + C were **b** is the slope and **C** is the intercept.
From regression analysis,

$$b = \frac{n\sum v.\frac{t}{v} - \sum v \sum \frac{t}{v}}{n\sum v^2 - (\sum v)^2} = \frac{7\ X\ 151112.98 - 0.8053\ X\ 1049497.6}{7\ X\ 0.1168 - 0.8053^2} = \frac{1057790.86 - 845160.41}{0.8176 - 0.6485}$$

$$\frac{212630.45}{0.1691} = 1257424.31 s/m^3$$

And Intercept C = $\frac{\frac{t}{v}}{n} - b\frac{v}{n}$ = 149928.23 – 144657.69 = 5270.54 s/m^3

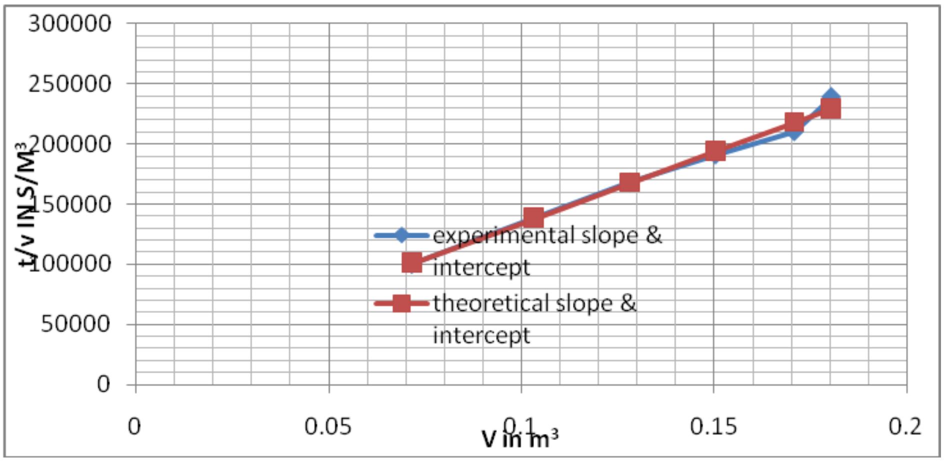

Fig 1: correlation between theoretical and experimental plot of t/v versus v at variable pressure

(Experimental slope =1260913.48 s/m^6 , Intercept = 4872.53 s/m^3)
(Theoretical slope = 1257426.75 s/m^6, Intercept = 5270.26 s/m^3).

The graph of the correlation between experimental and theoretical slopes and intercepts used to validate the modified equation are also displayed in Figures 2-4

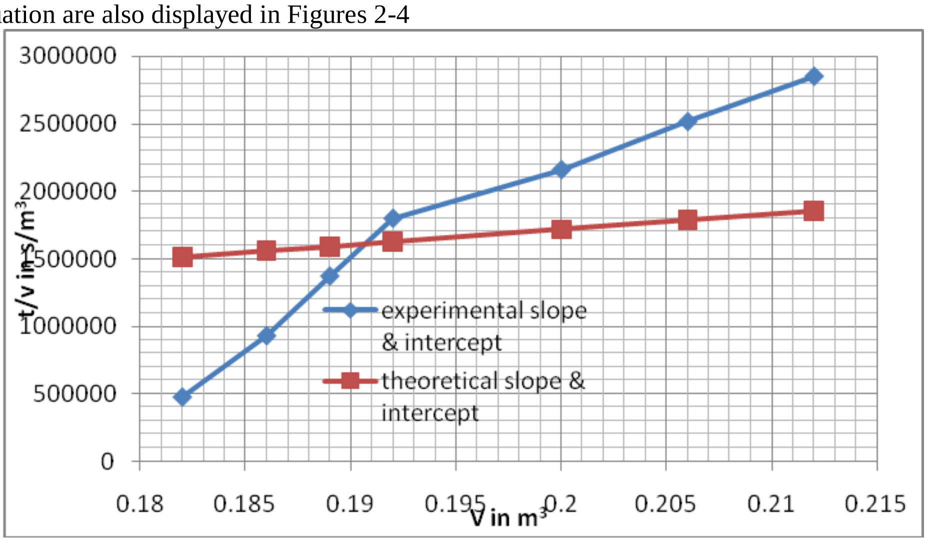

Fig 2: correlation between theoretical and experimental plot of t/v versus v at variable pressure
(Experimental slope =5359604.57 s/m^6, Intercept = 844882.56 s/m^3)
(Theoretical slope = 4579418.42 s/m^6, Intercept = 905658.24 s/m^3)

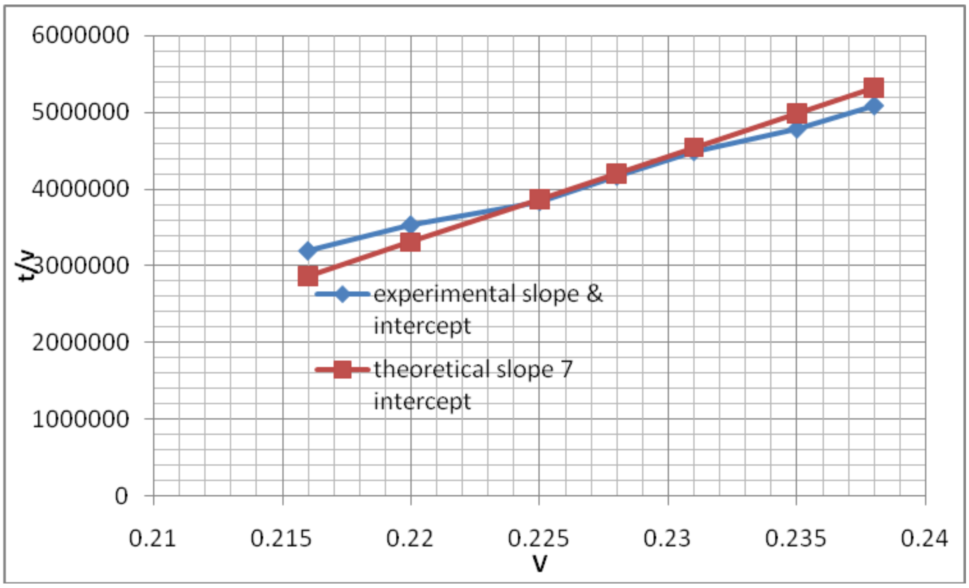

Fig 3: correlation between theoretical and experimental plot of t/v versus v

(Experimental slope = 112117050.4 s/m^6, Intercept = -2135816.16 s/m^3
(Theoretical slope = 112117075 s/m^6, Intercept = -21358166.74 s/m^3

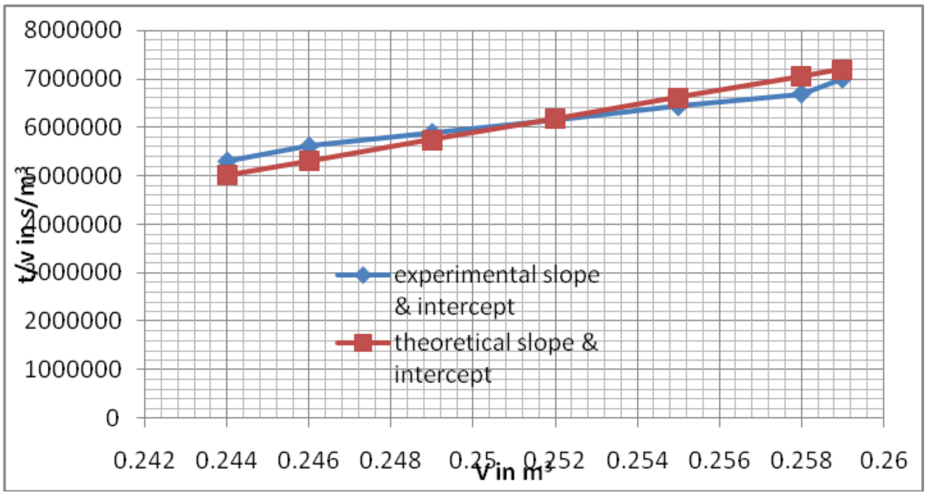

Fig 4: correlation between theoretical and experimental plot of t/v versus v
(Experimental slope = 145562880 s/m^6, Intercept = -30497917.03 s/m^3
(Theoretical slope = 206699290.5 s/m^6, Intercept = -4589555.58 s/m^3)

3.1 THE EFFECT OF FERRIC CHLORIDE ON SPECIFIC RESISTANCE

The data from the filtration experiment to investigate the effect of the weight of Ferric chloride conditioner on the specific resistance is recorded in table 3

Time t (s)	**Volume of filtrate (V) m^3**	**t/v**	**V^2**	**V.t/v**
1200	0.05118	23446.65	0.002619	1200
2400	0.05470	43875.69	0.002992	2400
3600	0.05604	64239.83	0.003140	3600
4800	0.05710	84063.05	0.003260	4800
6000	0.05720	104895.1	0.003271	6000
∑	**0.2762**	**320520.32**	**0.01528**	**18000**

Table 3: Data for filtration experiment using 10g of Ferric Chloride

Volume of sludge V = $0.063m^3$, Initial Hydrostatic Pressure P_{AV} = 684.3 N/m^2, Average Hydrostatic Pressure P $=\rho gh$ = 210.18N/m^2, Initial Height of sludge H_S = 0.07m, Average Height of sludge (H_{AV}) =0.014m, Temp.= 26°c, Density of water = 996.23kg/m^3, Area (A) = 0.9 m^2, Dynamic Viscosity μ = 0.892$N.s/m^2$, Weight of dry Solid W_d = 0.00941kg, Initial solid content (M) =0.149kg/m^3, Percentage of solid expressed in decimal P_S = 0.09, Specific gravity of sludge S_{sl} = 1.05,

By regression analysis, $b = \dfrac{n\sum v.\frac{t}{v} - \sum v \sum\frac{t}{v}}{n\sum v^2 - (\sum v)^2}$

$$= \frac{5 X 18000 - 0.2762 X 320520.32}{5X0.01528 - 0.0763} = \frac{90000 - 88527.71}{0.0764 - 0.0763} = 14722900 \text{ s/m}^6$$

$$C = \frac{\frac{t}{v}}{n} - b\frac{v}{n} = 64104.06 - 813292.99 = 749188.94 \text{ s/m}^3$$

Solving for the specific resistance and the compressibility coefficient of the sludge we have from

$$\frac{t}{v} = \frac{V\mu W_d R P_s}{P_I H_s A^3} + \frac{\mu W_d R P_s S}{A^2}$$

$$R = \frac{A^3 P_I b H_S}{\mu W_d P_S} = \frac{0.9^3 X684.3 X 0.07 X 14722900}{0.892 X 0.00941 X 0.09X} = 6.805632338X 10^{11} \text{ m/Kg}$$

$$S = \frac{CA^2}{\mu W_d P_S R} = \frac{749188.94 X 0.9^2}{0.892 X 0.00941 X 0.09 X 6.805632338 X10^{11}} = 0.00118 \text{ M}^2\text{/KN}$$

From the result, the specific resistance was determined to be 6.805632338 x 10^{11} m/Kg.

The values of specific resistance for four other filtration experiment using 20g, 30g , 40g, and 50g of Ferric chloride dosage yields $5.034388894 \times 10^{11}$ m/kg , $4.221393301 \times 10^{11}$ m/kg, 3.830709783X 10^{11} m/kg, and 1.65123474 $\times 10^{11}$ m/k and the variations of t/v versus v for the different weight of ferric chloride conditioning is shown below:

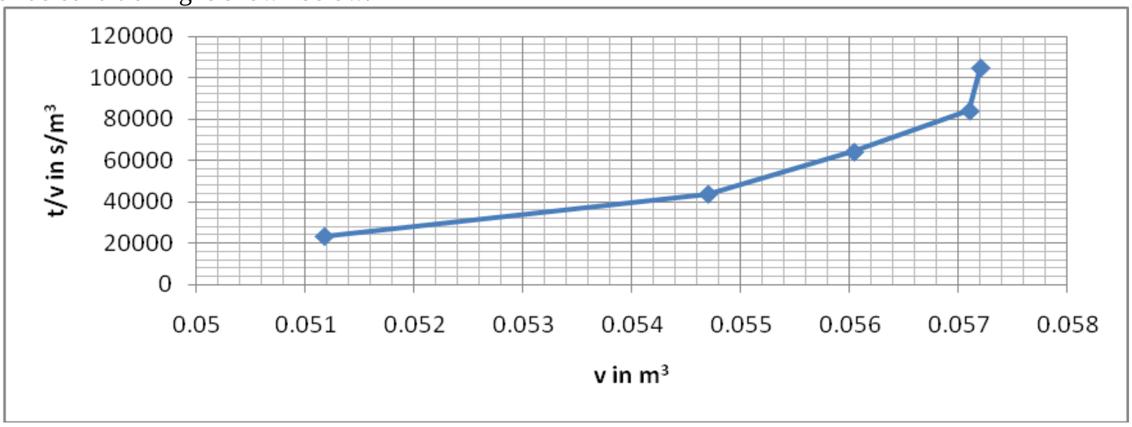

Fig 5: variation of t/v versus v for 10g of Ferric Chloride Conditioner

Amount of Ferric Chloride (g)	**Specific Resistance** (m/Kg)
10g	6.805632338 X 10^{11}
20g	$5.034388894 \times 10^{11}$
30g	$4.221393301 \times 10^{11}$
40g	3.830709783 X 10^{11}
50g	1.65123474 X 10^{11}

Table 4: Variation of ferric chloride Dosage with Specific Resistance

3.2 VARIATION OF SPECIFIC RESISTANCE WITH FERRIC CHLORIDE DOSAGE

The effect of conditioning $0.063m^3$ of digested sludge with varying weights (10g, 20g, 30g, 40g, 50g) of ferric chloride shows that specific resistance reduces as the weight of ferric chloride increases. This is so because as the chemical conditioner increases, the sludge particles become more loosely held increasing the porosity and hence reducing the resistance to fluid passage.

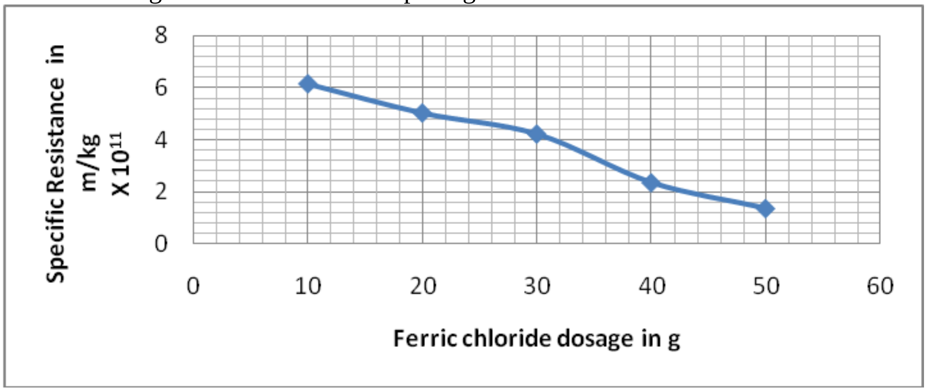

Fig 6: Variation of specific resistance with Ferric chloride dosage

3.3 VARIATION OF SPECIFIC RESISTANCE WITH INITIAL SOLID CONTENT

The result of the plot of specific resistance against the Initial solid content is displayed below in figure 7. From the graph initial solid content increases with decrease in specific resistance which is in agreement with the discovery made by coackley, (1956) in which he explained that the reduction in specific resistance with increase of initial solid content was due to variation in the state of digestion of the sludge particles.

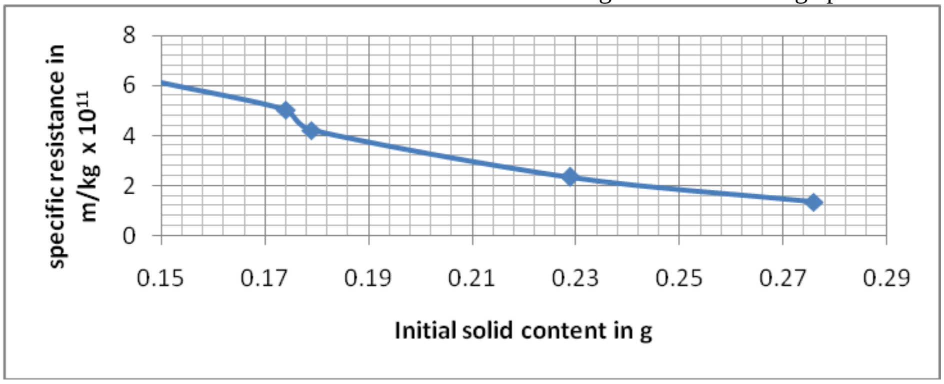

Fig 7: Variation of specific resistance with Initial solid content **(M**

3.4 VARIATION OF SPECIFIC RESISTANCE WITH PRESSURE

Figure 8 below shows the graph of specific resistance against pressure. From the graph, it can be seen that as pressure increases, the specific resistance also increases. This is because as time elapses during filtration more suspended solids will settles on top of the sand bed reducing the porosity of the sludge particles to the extent that the resistance to the flow of water through the sludge particles gradually increases as the water tries to flow through it.

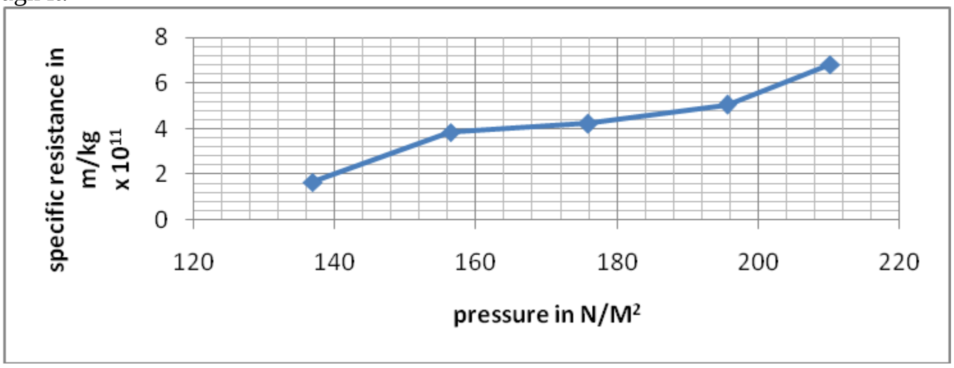

Fig 8: Variation of Specific Resistance with pressure

3.5 DETERMINATION OF COMPESSIBILITY COEFFICIENT

The result of laboratory experiment to determine the variation of void ratio with pressure by using digested sludge conditioned with 10g of Ferric chloride solution is shown below in table 5. The data obtained was used to compute compressibility coefficient of the sludge and is shown in figure 9. The details of the analysis are given thus:.

Height of ring = 6cm, Diameter of ring = 1.8cm

Area of ring (A) = $\pi d^2/4$ = 28.3cm, Specific gravity of sludge (S_{Sl}) = 1.05

Mass of ring + wet sludge in oven = 63.4g, Mass of dry sludge (W_d) = 20.5g

$H_S = \frac{W_d}{A\, S_{sl}\, \rho_w}$ = 0.68cm

Applied pressure P N/cm^2	**Final Dial reading At end of compression (mm)**	**Change in Dial Reading ΔH (mm)**	**Specimen height H at end of compression H = H_1 + ΔH (mm)**	**e = $\frac{H-H_s}{H_s}$ H_S = 6.8(mm)**
0	0.2		20	1.94
		(-) 0.11		
5	0.31		19.89	1.925
		(-) 0.23		
10	0.54		19.66	1.891
		(-) 0.75		
20	1.29		18.91	1.781
		(-) 0.41		
40	1.7		18.50	1.721
		(-) 0.14		
80	1.84		18.36	1.70

Table 5: Determination of Void ratio using sludge conditioned with 10g ferric chloride

Applied pressure P (KN/m^2)	**Change in Pressure ΔP (KN/m^2)**	**Change in Δ e**	**Compressibility Coefficient $\frac{\Delta e}{\Delta P}$ (cm^2/KN)**	**Mean of compressibility coefficient (cm^2/KN)**
	5	0.015	0.003	
5				
	5	0.034	0.007	
10				
	10	0.11	0.011	
20				
	20	0.06	0.003	
40				
	40	0.02	0.0005	**0.004**

Table 6: Compressibility coefficient from sludge conditioned with 10g ferric chloride

The laboratory procedure as used above was also used to compute the compressibility coefficient for four other experiments using 20g weight, 30g weight, 40g weight and 50g weight and having compressibility coefficients of 0.005 **cm^2/KN**, 0.006 **cm^2/KN**, 0.007 **cm^2/KN**, and 0.005 **cm^2/KN** respectively.

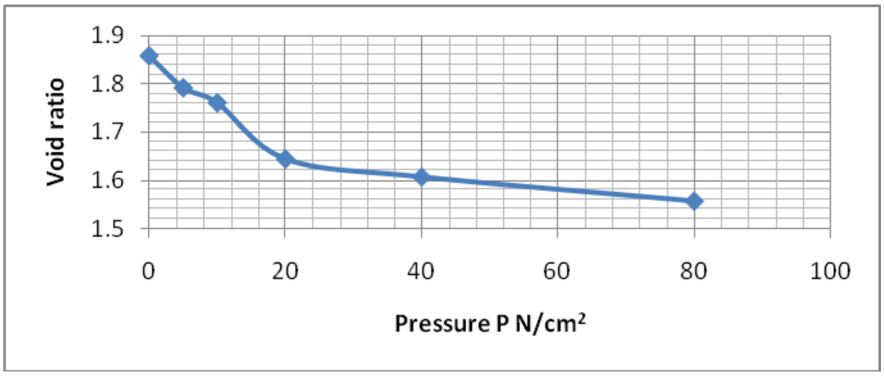

Fig 9: Variation of void ration with pressure.

IV. CONCLUSION

The application of a modified $FMTL_XL_YL_Z$ dimensional analysis technique to the derivation of sludge filtration equation using sand drying bed has been presented. The modified equation incorporates the compressibility coefficient believed to affect sludge dewatering phenomenon. The equation was verified using data from the filtration experiment of digested sludge collected from the oxidation pond of the University of Nigeria, Nsukka and there was a close agreement between experimental and theoretical values with the correlation coefficient ranging from (0.94-0.98). For the experimental plot the slopes and intercept are (1260913.48 s/m^6 , 4872.53 s/m^3) , (5359604.57 s/m^6, 844882.56 s/m^3), (112117050.4 s/m^6, -2135816.16 s/m^3), and (145562880 s/m^6, -30497917.03 s/m^3) while the theoretical values of slopes and intercepts are (1257426.75 s/m^6, 5270.26 s/m^3),(4579418.42 s/m^6, 905658.24 s/m^3), (112117075 s/m^6, -21358166.74 s/m^3),and (206699290.5s/m^6,-4589555.58s/m^3) respectively.;

- This research has illustrated the application of a modified $FMTL_XL_YL_Z$ dimensional analysis technique in deriving filtration equation through the use of sludge drying bed.
- The equation derived, unlike many others, shows a close agreement between experimental and theoretical values.
- It has shown the performance of the natural sand drying bed filtration in the operation of sludge filtration process.
- It has also shown that sludge is a compressible material and so its compressibility attributes must be accounted for in any filtration equation if that equation is to be accepted by the body of researchers.

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LISFT OF SYMBOLS

A = Cross sectional area (m^2)
V = Volume of filtrate m^3
T = Time of filtration (s)
H = Driving Head of sludge (m)
ΔH = Change in sludge height
R = Specific resistance (m/kg)
S = Compressibility coefficient (m^2/KN)
C = Intercept on the t/v axis (s/m^3)
b = Slope of t/v versus v (s/m^6)
Δe = Change in void ratio
.e = Void ratio
P_1 = Initial pressure (KN/M^2)
W_d = Weight of dry sludge (kg)
P_S = Percent of solid content expressed in decimal
M = Initial solid content (kg)
V_{sl} = Volume of sludge (M^3)
S_{sl} = Specific gravity of sludge
H_s = Initial sludge height (m)
$FMTL_XL_YL_Z$ = Modified dimensional system